Displacing Sensors: Coverage and Interference Problems

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Why Displacement for Coverage and Interference?

- Consider a network of sensors employed in a terrain.
- In unfavourable terrains (e.g., presence of obstacles)
  - coverage may be disturbed.
- Proximity may cause interference.
- Network connectivity/recovery requires that sensors must be moved from their initial positions.
- There is a tradeoff between number of sensors and movement:

  If you want to save on the number of sensors used you will have to pay on the sensor movement and vice versa if you want to save on the sensor movement you will have to pay on the number of sensors!

Outline

- Motivation
- Related Work
- Model
- Interference
  - Results
  - Main Ideas
- Coverage
  - Results
  - Main Ideas

Motivation

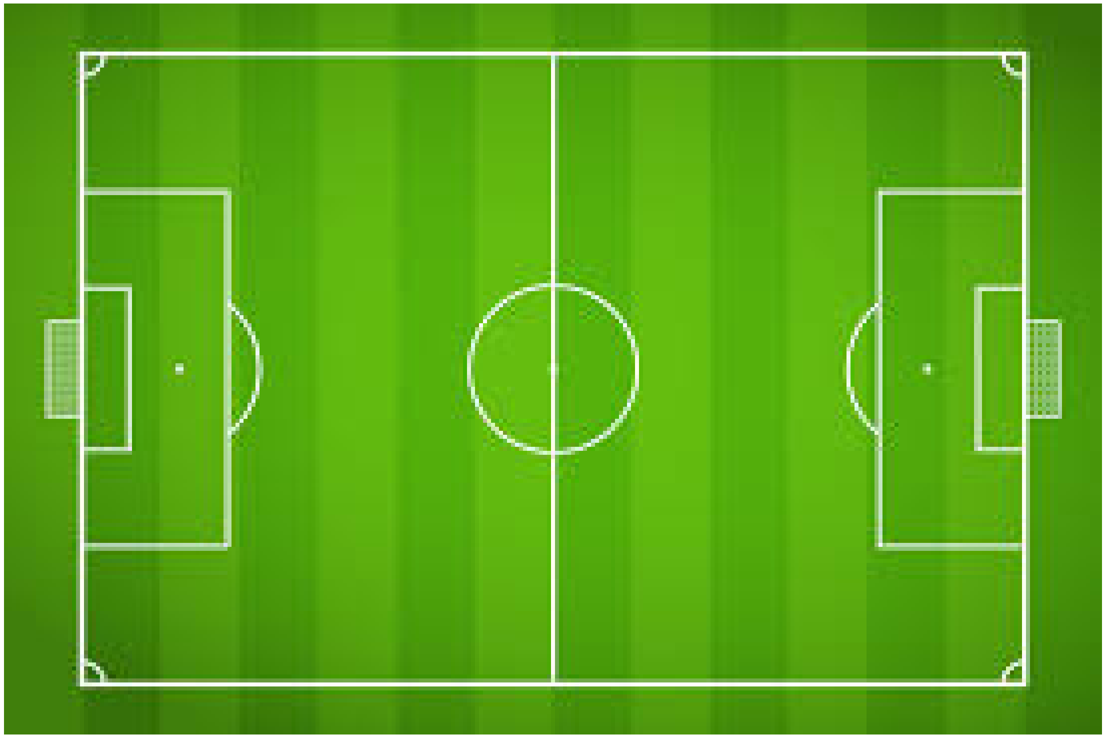
Why Monitoring: Sensors in a Vineyard

- Making Canadian "Ice Wine".
- Very sensitive to temperature changes.

- … harvest late in the fall season and wait for the temperature to drop to $-7C$!

The Most Important Questions in ihe Beautiful Game!

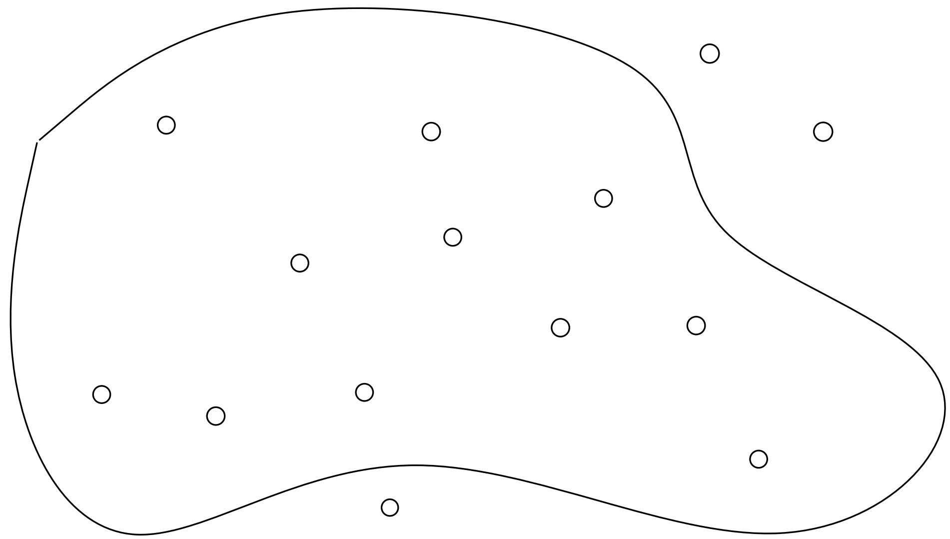
- Did the Ball Cross the Line?



- Which Line?
- When?

Sensor (Barrier) Coverage

- A geometric domain and n sensors in(out)side the domain.



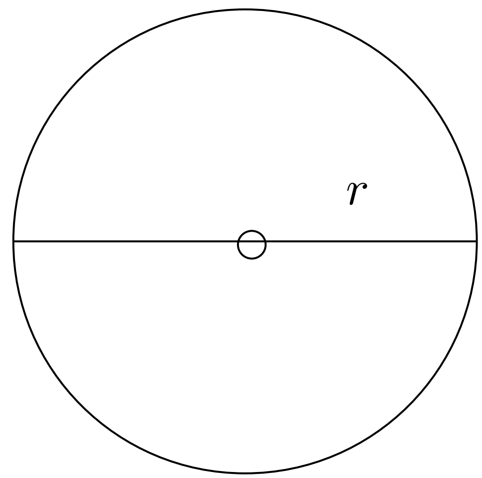
- Sensors may not cover the (barrier of the) domain!
- We want to cover the (barrier of the) domain in the sense that every point in the (barrier of the) domain is within the range of a sensor.

Coverage vs Barrier Coverage of a Domain

- There is a slight difference:
  - In *coverage* the whole area of the domain needs to be monitored;
  - In *barrier coverage* only the boundary domain needs to be monitored.
- If your goal is to monitor intruders the latter can be accomplished more efficiently than the former.

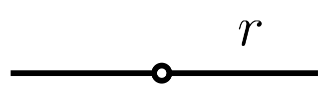
Coverage from a Sensor of Range r

- When coverage is on the plane:



the sensor covers a disc of radius r.

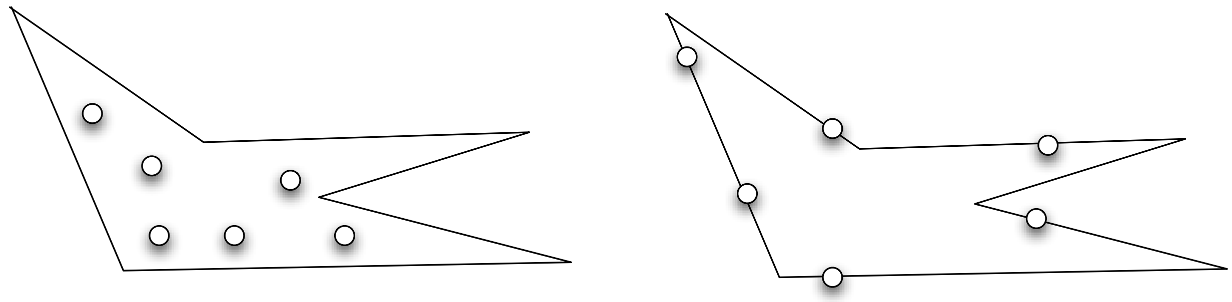
- When coverage is on a line:



the sensor covers a segment of length $2r$.

Questions: Movement for (Barrier) Coverage

- Geometric domain with a well-defined boundary; n sensors.

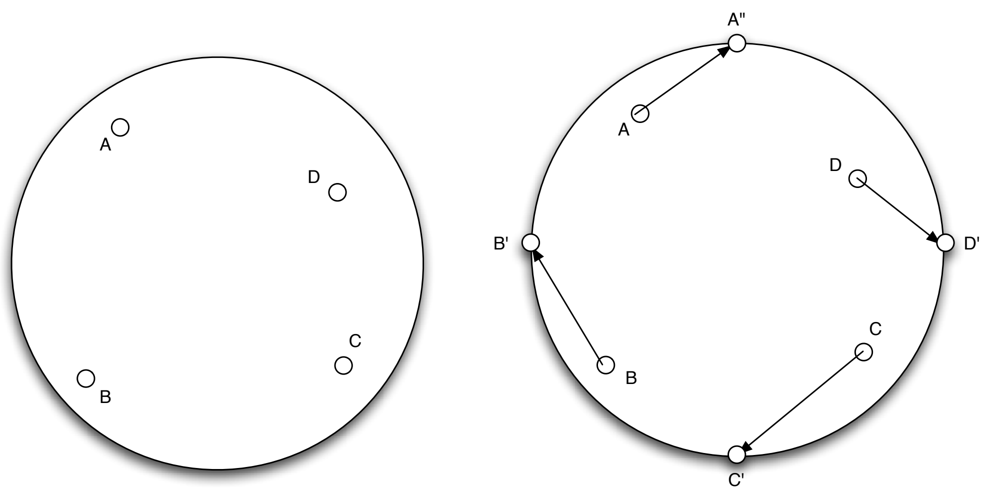


- **Question:** Do the sensors cover the domain?
- **Question:** Do the sensors cover the boundary?
- **Question:** If not, move the sensors from their original positions to new positions to accomplish the task?
- **Main Question:** What's the optimal cost of displacement?

NB: Cost can be distance, some power of distance, time, etc!

Main Optimization Problem(s)

- Assume $d(A,\ B)$ measures (some) cost from A to B.
- **Sensor Displacement:** when n sensors at initial positions $A_1, A_2, \ldots, A_n$ move to new positions $A'_1, A'_2, \ldots, A'_n$, the *total displacement* cost is $\sum_{i=1}^{n} d(A_i, A'_i)$, and

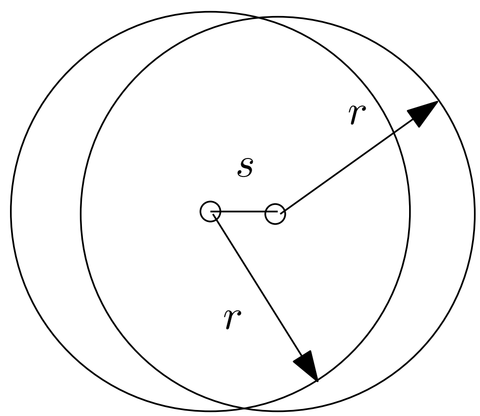


the *max displacement* cost is $\max_{i=1}^{n} d(A_i, A'_i)$.

- **Optimization Problem:** Minimize sum (or max) over all new positions $A'_1, A'_2, \ldots, A'_n$ which accomplish (barrier) coverage.

Proximity and Sensor Interference

- Proximity between sensors affects their transmission and reception signals and degrades network performance.
  - *The closer their distance the higher the resulting interference and hence performance degradation.*
- A critical value, say $s > 0$, is established and sensors are kept a distance of at least s apart.



  - *Signals interfere during communication if distance is* $< s$.

Related Work

Communication and Movement Algorithms (1/2)

- Deterministic Input
  - How efficiently can you move the sensors?
    - ∗ Minimize the energy
    - ∗ Minimize the time
    - ∗ Minimize the number of sensors moved
  - How do sensors communicate?
    - ∗ Global
    - ∗ Local
- Some Recent Research in
  - MOBICOM 06, COCOA 08 (TCS 09), ADHOCNOW 09 & 10, WADS 2012, PODC 13, ...

Communication and Movement Algorithms (2/2)

- Random Input
  - Type of distribution
  - Relationship of sensor range and movement

References

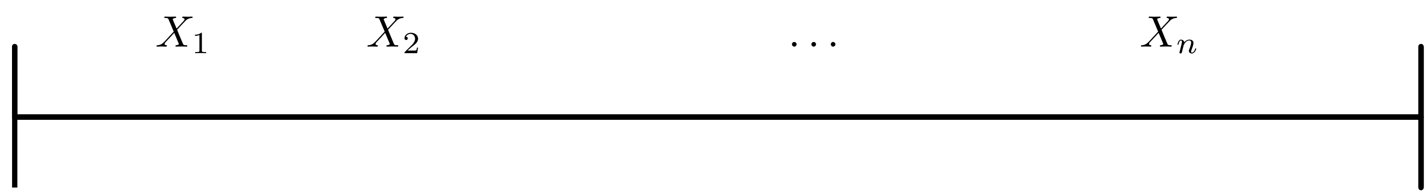
E. Kranakis and G. Shaikhet. Displacing sensors to avoid interference. In *Proceedings of 20th COCOON*, 2014.

E. Kranakis, D. Krizanc, O. Morales-Ponce, L. Narayanan, J. Opatrny, and S. Shende. Expected sum and maximum of displacement of random sensors for coverage of a domain. In *Proceedings of the 25th SPAA*, pages 73–82. ACM, 2013.

Model

Random Model for Coverage

- Coverage Problem in the unit interval $[0, 1]$:

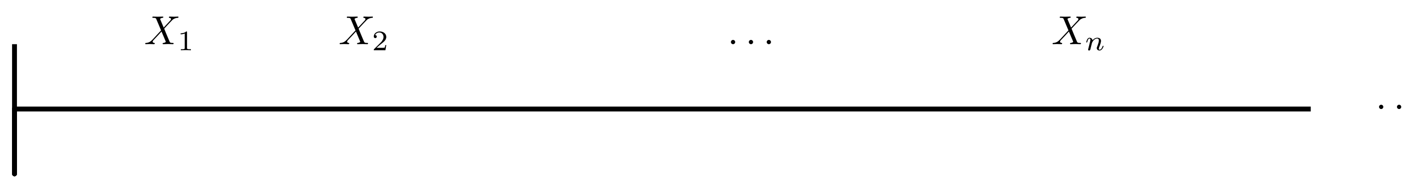


Sensors are thrown randomly and independently with the uniform distribution in the unit interval.

- $X_1, X_2, \ldots, X_n$ represent sensor positions.

Random Model for Interference

- Interference Problem in the half-line $[0, +\infty)$:



- X_i is the i-th arrival in a Poisson process.

Coverage: Motivation (1/3)

- Throw n sensors of radius $r := \frac{1}{2n}$ at random in a unit interval.
- To ensure coverage of the interval they must be moved to anchors $a_i = \frac{i}{n} + \frac{1}{2n}$, for $i = 0, 1, \ldots, n-1$.
  - This is the worst-case total movement!
  - The cost is roughly $\sqrt{n}$.
  - Why?
- Do a simulation!

Coverage: Motivation (2/3)

- Keep increasing the sensor radius.
  - The bigger the radius the less the movement! Why?
- For n sensors of radius $\Theta(\frac{\ln n}{n})$, w.h.p. no sensor needs to move!
- Why?
  - The probability that no sensor drops inside a subinterval of length x is $(1-x)^n$.

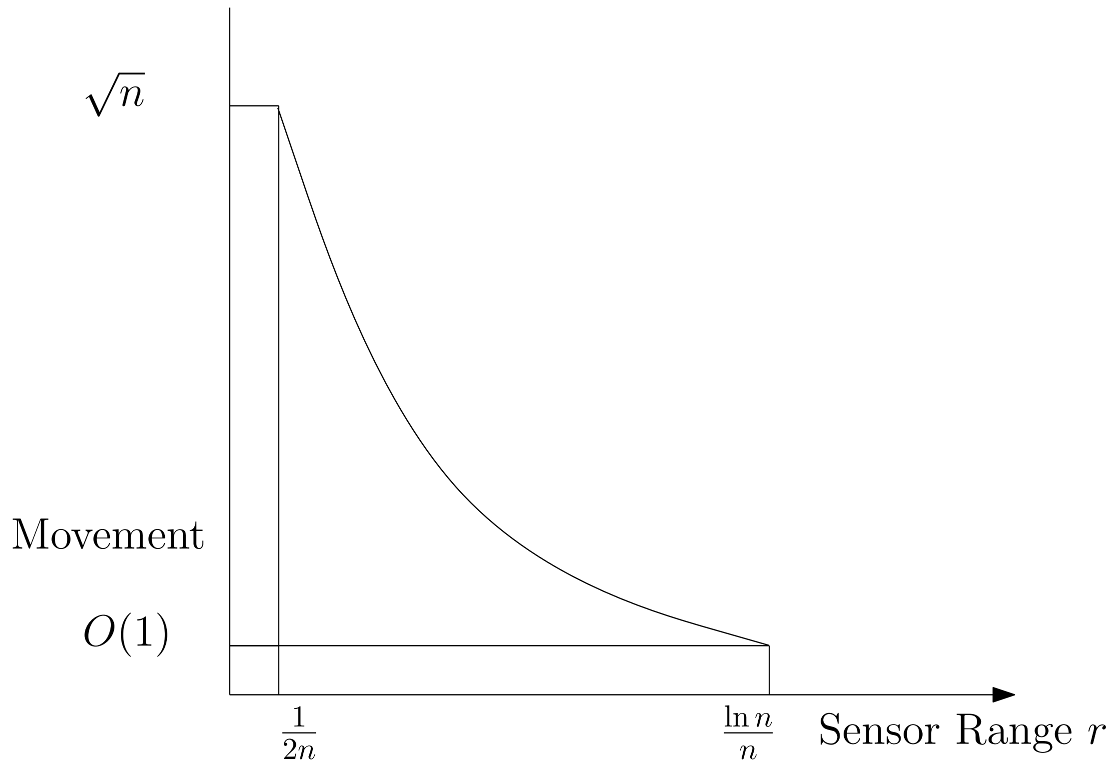
  - However,

$$(1-x)^n = \left(1 - \frac{xn}{n}\right)^n \approx e^{-xn} = \frac{1}{n^c},$$

for $x = \frac{c \ln n}{n}$, where $c > 0$.

Coverage: Prediction (3/3)

- Sensor movement as a function of the sensor range.



- The bigger the radius (range) the smaller the movement.

Interference: Motivation (1/2)

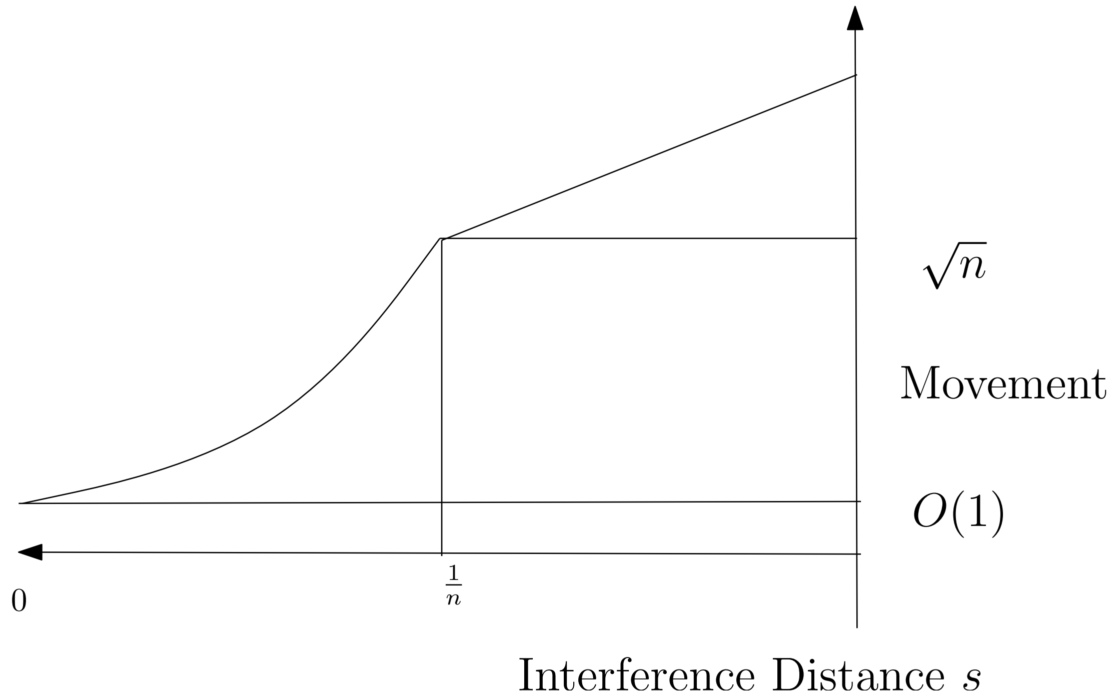
- Throw n sensors at random in a unit interval. We want to ensure no two sensors are at distance $< s$.
  - To ensure no two sensors are at distance $< \frac{1}{2n}$ they must all be placed to anchors $a_i = \frac{i}{n} + \frac{1}{2n}$, for $i = 0, 1, \ldots, n-1$. This is the worst-case total movement! Why?
- Keep decreasing the interference distance s.
  - The smaller the interference distance s the less the movement! Why?
- In general,

$$\text{Arrival Time of } i+1\text{st sensor} - \text{Arrival Time of } i\text{th sensor}$$

are the interarrival times of the Poisson process.

Interference: Prediction (2/2)

- Sensor movement as a function of the sensor distance.



- The smaller the interference distance the smaller the movement.

Interference

Displacement and Interference on a Line

- Assume that n sensors arrive according to a Poisson process having arrival rate $\lambda = n$ in the interval $[0, +\infty)$.
  - What is the expected minimum total distance that the sensors have to move from their initial position to a new destination so that any two sensors are at a distance more than s apart?

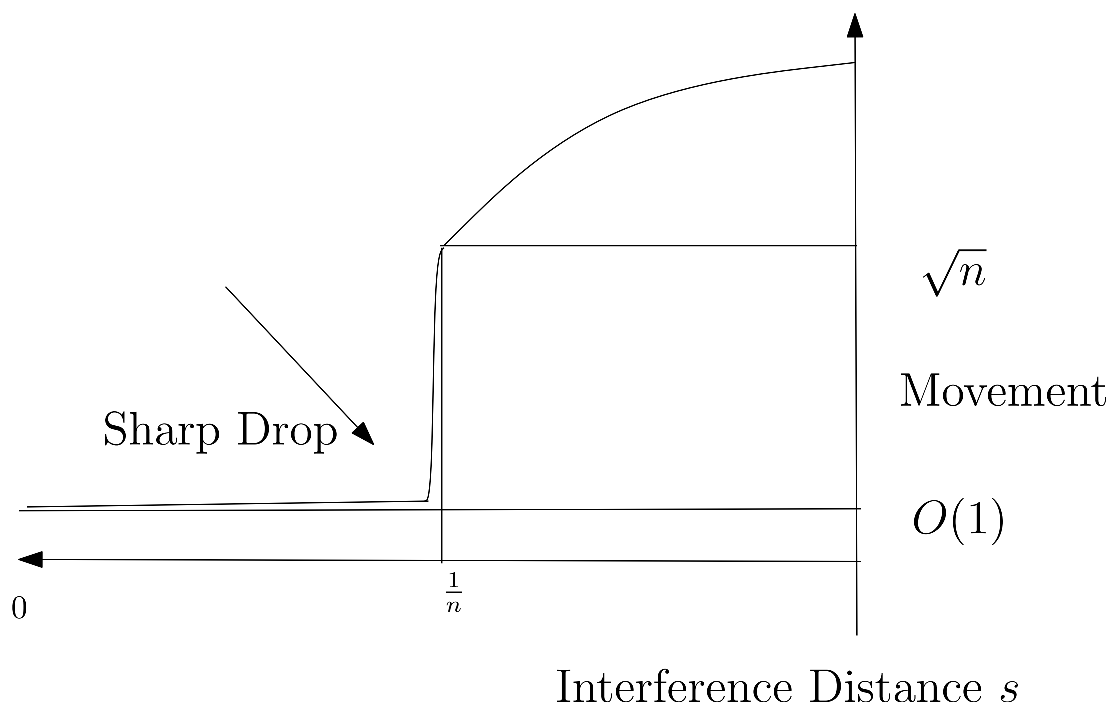
Results on Interference

- We obtain tradeoffs between the interference distance s and the expected minimum total movement, denoted by $E(s)$ (see Kranakis and Shaikhet [2014]).

Interference Distance s	Total Displacement $E(s)$
$s - \frac{1}{n} \in \Omega\left(n^{-\alpha}\right), 2 \geq \alpha \geq 0$	$\Omega(n^{2-\alpha})$
$\left\|s - \frac{1}{n}\right\| \in O\left(n^{-3/2}\right)$	$\Theta(\sqrt{n})$
$s \leq \frac{1}{tn}, t > 1$	$\leq \frac{t^2}{(t-1)^3}$

Critical Regime

- On a line there is **critical threshold** around $\frac{1}{n}$,
  1. for s below $\frac{1}{n} - \frac{1}{n^{3/2}}$, $E(s)$ is a constant $O(1)$,
  2. for $s \in \left[\frac{1}{n} - \frac{1}{n^{3/2}}, \frac{1}{n} + \frac{1}{n^{3/2}}\right]$, $E(s)$ is in $\Theta(\sqrt{n})$,
  3. for s above $\frac{1}{n} + \frac{1}{n^{3/2}}$, $E(s)$ is above $\Theta(\sqrt{n})$.
- Sensor movement as a function of the sensor distance.



Main Ideas

For s above $\frac{1}{n} + \frac{1}{n^{3/2}}$, $E(s)$ above $O(\sqrt{n})$

- The proof of this result is based on the following:

Theorem 1 *Assume that the interference value between sensors is s.*

1. *If $s - \frac{1}{n} \in \Omega(n^{-\alpha})$ then $E(s) \in \Omega(n^{2-\alpha})$, where $2 \geq \alpha \geq 0$ is a real number.*
2. *In particular, if $s - \frac{1}{n} \in \Omega(n^{-3/2})$ then $E(s) \in \Omega(\sqrt{n})$.*

Basic Idea of Proof (1/2)

- Let the r.v. D_i define the i-th sensor's displacement.
- After the sensors move to their final destinations it must be true that
$$D_{i+1} + X_{i+1} \geq D_i + X_i + s,$$
for all $1 \leq i \leq n-1$, so as to ensure that the two sensors are at least distance s apart.
- It follows that
$$E[D_{i+1}] + E[X_{i+1}] \geq E[D_i] + E[X_i] + s.$$
- However, $E[X_{i+1}] = \frac{i+1}{n}$ and $E[X_i] = \frac{i}{n}$.
- Therefore
$$E[D_{i+1}] \geq E[D_i] + s - \frac{1}{n},$$
for $1 \leq i \leq n-1$.

Basic Idea of Proof (2/2)

- Repeating this inequality recursively we see that

$$E[D_{i+1}] \geq E[D_1] + \left(s - \frac{1}{n}\right) i,$$

for $1 \leq i \leq n-1$.

- So, the expected minimum total movement must satisfy

$$\begin{aligned} \sum_{i=1}^{n} E[|D_i|] &\geq \sum_{i=1}^{n} E[D_i] \\ &\geq nE[D_1] + \left(s - \frac{1}{n}\right) \frac{n(n-1)}{2}. \end{aligned} \tag{1}$$

- Result follows easily from the observation $|nE[D_1]|$ is in $O(\ln n)$.

For s below $\frac{1}{n} - \frac{1}{n^{3/2}}$, $E(s)$ in $O(1)$ (1/2)

- The basic algorithm is the following:

Algorithm 1: Moving Sensors

1.	Set $M_1 = 0$;
2.	**for** $i = 2$ **to** n **do**
3.	move sensor X_i to new position $X_i + M_i$ s.t.
4a.	$X_{i-1} + M_{i-1} \leq X_i + M_i$;
4b.	$X_i + M_i \leq s + X_{i-1} + M_{i-1}$;

For s below $\frac{1}{n} - \frac{1}{n^{3/2}}$, $E(s)$ in $O(1)$ (2/2)

- The proof of this result is based on the following:

  Theorem 2 *Assume the interference distance between sensors is s. If $s \leq \frac{1}{tn}$ then*

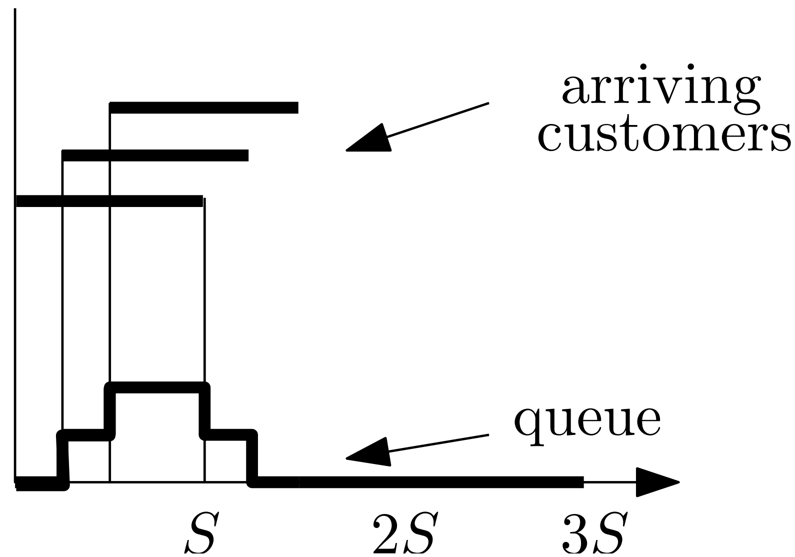
  $$E(s) \leq \min\left\{\frac{t^2}{(t-1)^3}, \frac{n-1}{2t}\right\}, \tag{2}$$

  where $t > 1$.

  – This is basically a result about $M/D/1$ queues: M is the Poisson arrival rate, D is deterministic (fixed sensor radius).

Basic Idea (1/2)

- The queue



Basic Idea (2/2)

- The upper bound $\frac{t^2}{(t-1)^3}$ uses queueing theory:
  - We have a single-server service station in which customers arrive according to a Poisson process having rate λ.
  - Arriving customer served immediately if server is free; and if not, customer joins the queue
  - Busy period begins when an arrival finds system empty.
  - Distribution of length of a busy period will be the same for each such period.
  - B is the r.v. denoting the length of a busy period, and S the service time of the first customer in the busy period.

For $s \in \left[\frac{1}{n} - \frac{1}{n^{3/2}}, \frac{1}{n} + \frac{1}{n^{3/2}}\right]$**,** $E(s)$ **in** $\Theta(\sqrt{n})$

- The proof of this result is based on the following:

  Theorem 3 (Critical Regime) *Assume the interference value between sensors is s. If* $\left|s - \frac{1}{n}\right| \in O\left(n^{-3/2}\right)$ *then* $E(s) \in \Theta(\sqrt{n})$.

Coverage

Displacing for Coverage in $[0, 1]$

- n sensors with identical range $r = \frac{f(n)}{2n}$, for some $f(n) \geq 1$, for all n, are thrown randomly and independently with the uniform distribution in the unit interval $[0, 1]$.

- They are required to move to new positions so as to cover the entire unit interval in the sense that every point in the interval is within the range of a sensor.

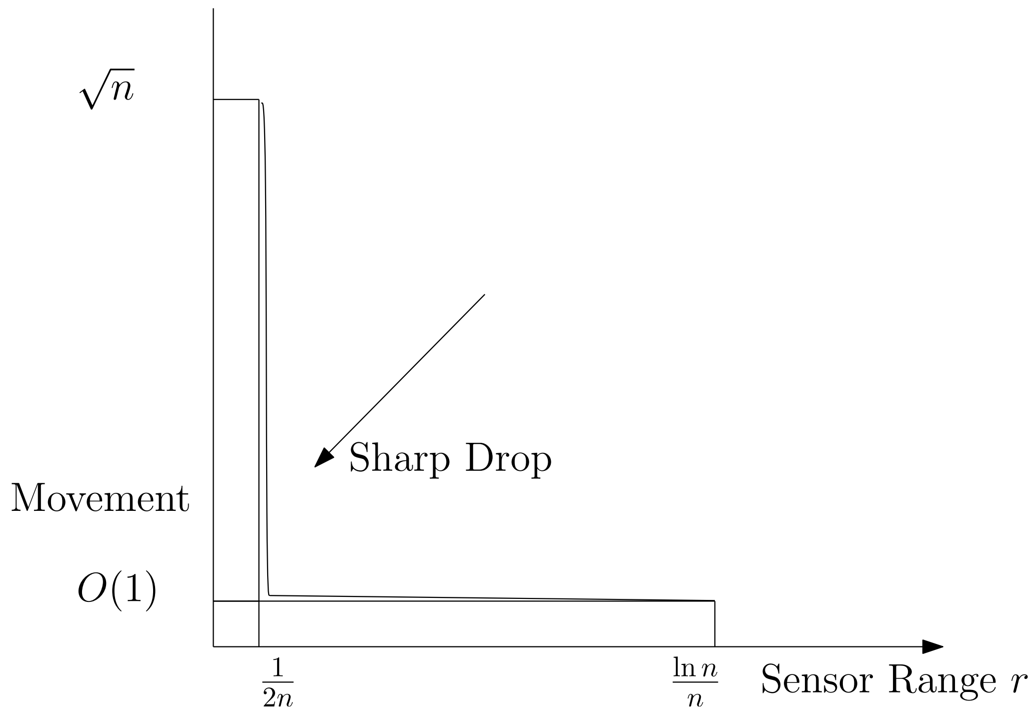
- We obtain tradeoffs (see Kranakis et al. [2013]) between the range r of the sensors and

  – the expected min sum (denoted by $E(r)$)

  of displacements of the sensors required to accomplish this task.

Results for the Unit Interval

Sensor Range r	Total Displacement $E(r)$
$\frac{1}{2n}$	$\Theta(\sqrt{n})$
$\frac{f(n)}{2n}$ $(f(n) \geq 6)$	$O\left(\sqrt{\frac{\ln n}{f(n)}}\right)$
$\frac{f(n)}{2n}$ $(12 \leq f(n) \leq \ln n - 2\ln\ln n)$	$O\left(\frac{\ln n}{f(n)e^{f(n)/2}}\right)$



Main Ideas

Displacing to Fixed Anchors

- First a tight bound on the displacement:

  Theorem 4 *Assume that n mobile sensors are thrown uniformly and independently at random in the unit interval. The expected sum of displacements of all n sensors to move from their current location to anchor locations $\frac{i}{n} - \frac{1}{2n}$, for $i = 1, \ldots, n$, respectively is $\Theta(\sqrt{n})$.*

Displacement Algorithm 1

- How do you move the sensors when $r = \frac{f(n)}{2n} \geq \frac{1}{2n}$?
- $\boxed{\textbf{Displacement Algorithm 1}}$
  1. Divide the interval into subintervals of length $6 \ln n/n$;
  2. If there is a subinterval with fewer than $\ln n$ sensors then use the standard algorithm that moves all n sensors to positions that are equidistant and stop;
  3. Otherwise, in each subinterval choose $\lceil 6 \ln n/f(n) \rceil$ ($\leq \ln n$ since $f(n) \geq 6$) sensors at random and move the chosen sensors to equidistant positions $f(n)/n$ apart;
- How good is this algorithm?

Length Contraction Lemma

- **Lemma 1 (Length Contraction)** *Assume that m sensors are thrown randomly and independently with the uniform distribution on an interval of length x. The sensors are to be moved to equidistant positions (within this interval) at distance x/m from each other. The total expected movement of the sensors is $\Theta(x\sqrt{m})$.*

Idea of Proof of Lemma 1

- Consider m sensors in the interval $[0, x]$ (of length x).
- Multiply their coordinates by $1/x$ to normalize the problem over the unit interval.
- By our previous result the total movement in the unit interval is in $O(\sqrt{m})$.
- Now “scale this back” by multiplying by x and we get $x\sqrt{m}$, which is the desired result.

Displacement due to Algorithm 1 (1/3)

- **Theorem 5** *Let* $r = f(n)/2n$ *where* $f(n) \geq 6$, *for all* n. n *sensors of radius* r *are thrown randomly and independently with uniform distribution on a unit interval. The total expected movement of sensors required to cover the interval is* $O(\sqrt{\ln n / f(n)})$.

Displacement due to Algorithm 1 (2/3)

- There are two cases to consider
- *Case 1:* There exists a subinterval with fewer than $\ln n$ sensors.
  - In this case the total expected movement is $O(\sqrt{n})$ by Theorem 4.

Displacement due to Algorithm 1 (3/3)

- *Case 2:* All subintervals contain at least $\ln n$ sensors.
- By the independence of sensor positions, the $6 \ln n / f(n)$ chosen sensors in any given subinterval are distributed randomly and independently with uniform distribution over the interval of length $6 \ln n / n$.
- By Lemma 1, the expected movement inside each interval is $O\left((\ln n/n) \sqrt{\ln n / f(n)} \right)$.
- There are $n/(6 \ln n)$ intervals and total expected displacement $O(\sqrt{\ln n / f(n)})$.
- Now use Chernoff bounds:

  Claim 1 *The probability that fewer than* $\ln n$ *sensors fall in any subinterval is* $< 1/n$.

An Improvement

- The above theorem can be improved for large enough radii using occupancy estimates:

  Theorem 6 *Let* $12 \le f(n) \le \ln n - 2 \ln \ln n$*, for all* n*. If* n *sensors of radius* $r = f(n)/2n$ *are thrown randomly and independently with the uniform distribution on a unit interval then the total expected movement of sensors required to cover the interval is* $O\left(\frac{\ln n}{f(n)e^{f(n)/2}}\right)$.

- The algorithm is as follows.

Displacement Algorithm 2

- $\boxed{\textbf{Displacement Algorithm 2}}$

1. Divide the interval into subintervals of length $\frac{6\ln n}{n}$;
2. If there is a subinterval with fewer than $\ln n$ sensors then use the standard algorithm that moves all n sensors to positions that are equidistant and stop;
3. Otherwise, divide subintervals into "bins" of size $\frac{f(n)}{2n}$;
   (a) If the total number of *empty* bins over all subintervals is greater than $\frac{4n}{f(n)e^{f(n)/2}}$ then use the standard moving algorithm as above and stop;
   (b) Otherwise, within each subinterval find a matching of extra sensors to empty bins and move the sensors accordingly;

Open Problems

- Displacement cost may depend on
  - terrain
  - sensor speed
- More realistic models of interference
- Stronger models of coverage
- 2D

Thank you!